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INTEGRATION OF A BIVARIATE NORMAL OVER AN OFFSET CIRCLE

By Marc Mangel, David B. Quanbeck

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TABLE OF CONTENTS

INTRODUCTION

There are many instances in operations research and statistics in which one must integrate a bivariate normal density over an offset circle. Namely, let

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)^{1/2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\right. \tag{1}$$

$$\left.\left[\left(\frac{x-m_x}{\sigma_x}\right)^2 - \frac{2\ (x-m_x)(y-m_y)}{\sigma_x\sigma_y} + \left(\frac{y-m_y}{\sigma_y}\right)^2\right]\right\} ,$$

be the density for a bivariate random variable (X,Y). The probability $P[R]$, that (X,Y) falls in the offset circle R defined by:

$$R = \{x,y: (x-x_c)^2 + (y-y_c)^2 \leq r^2\} \tag{2}$$

is

$$P[R] = \iint_R f(x,y)dxdy . \tag{3}$$

Equation 3 is the coverage integral or coverage function (e.g., see references 1 through 6).

The coverage integral has been calculated in certain special cases (references 6 and 7). When $m_x = m_y = 0$ and the circle is centered at the origin, one can convert the integrand to a Bessel function, expand in a power series and integrate term by term (reference 6).

If $\sigma_x = \sigma_y = \sigma$, then $P[R]$ can be given in terms of the non-central χ^2 distribution (reference 7).

The result of reference 6, however, converges very slowly if $r/\sqrt{2}\sigma_1 > 1$, where $\sigma_1 = \min(\sigma_x, \sigma_y)$. The result of reference 7 is not valid for non-circular densities.

In this paper, a formula is derived for the calculation of the coverage integral in the most general case. A sample problem is presented and in the appendix a Fortran code is listed for implementing the formula on a computer.

THE ITERATED INTEGRAL

With reference to figure 1, it is seen that equation 3 can be rewritten as:

$$P[R] = \int_{x_c-r}^{x_c+r} \int_{y_c-(r^2-(x-x_c)^2)^{1/2}}^{y_c+(r^2-(x-x_c)^2)^{1/2}} f(x,y)\,dy\,dx \tag{4}$$

$$= \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)^{1/2}} \int_{x_c-r}^{x_c+r} \int_{y_c-(r^2-(x-x_c)^2)^{1/2}}^{y_c+(r^2-(x-x_c)^2)^{1/2}}$$

$$\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-m_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-m_x}{\sigma_x}\right)\left(\frac{y-m_y}{\sigma_y}\right) + \left(\frac{y-m_y}{\sigma_y}\right)^2\right]\right\}dy\,dx. \tag{5}$$

Let $I(R)$ denote the integral in equation 5, so that:

$$P[R] = \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)^{1/2}}\, I(R)\ . \tag{6}$$

This integral can be rewritten as:

$$I(R) = \int_{x_c-r}^{x_c+r} \exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{x-m_x}{\sigma_x}\right)^2\right]\left\{\int_{y_c-(r^2-(x-x_c)^2)^{1/2}}^{y_c+(r^2-(x-x_c)^2)^{1/2}}\right.$$

$$\left.\exp\left[-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{y-m_y}{\sigma_y}\right)^2 - 2\rho\left(\frac{x-m_x}{\sigma_x}\right)\left(\frac{y-m_y}{\sigma_y}\right)\right\}\right]dy\right\}dx\ . \tag{7}$$

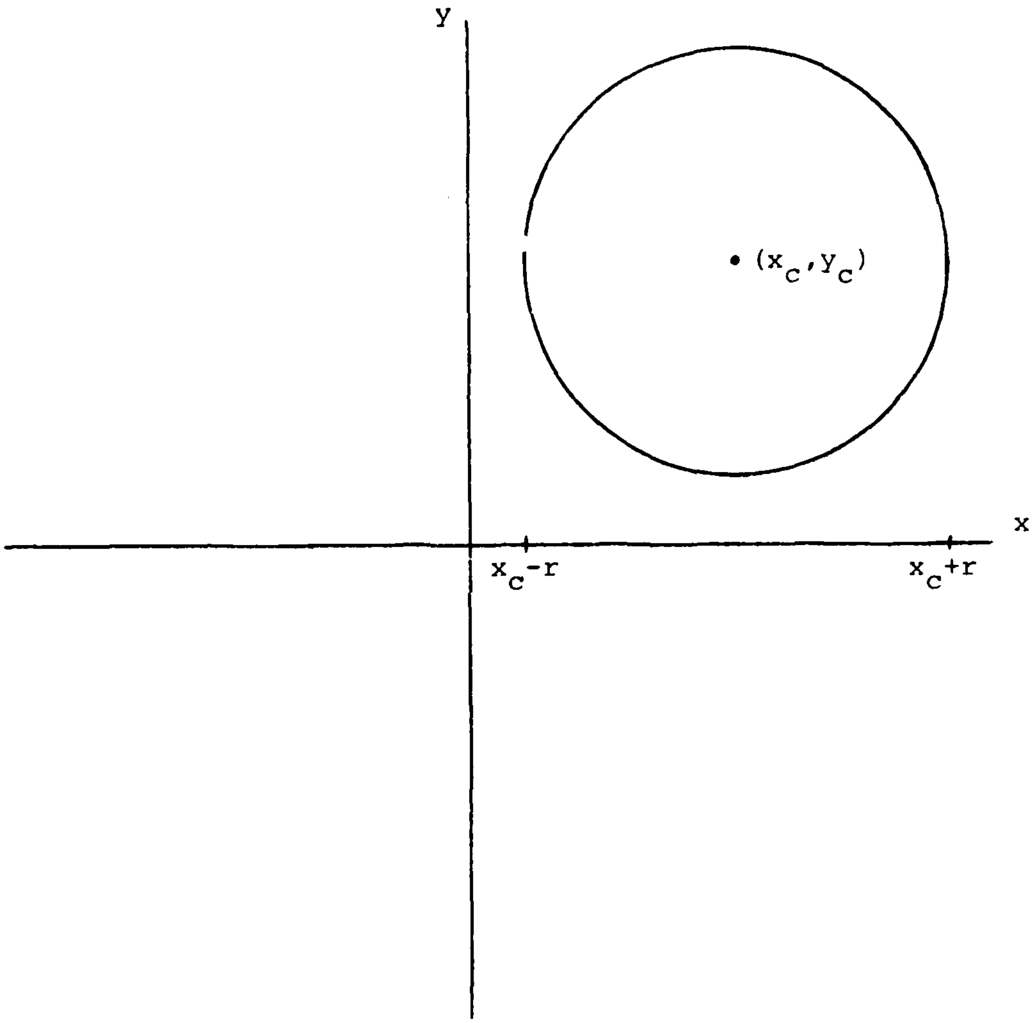

FIG. 1: THE OFFSET CIRCLE

First, consider the integral over the y-variable. The exponent of the integrand can be rewritten as:

$$\left(\frac{y-m_y}{\sigma_y}\right)^2 - 2\rho\left(\frac{x-m_x}{\sigma_x}\right)\left(\frac{y-m_y}{\sigma_y}\right) = \left[\left(\frac{y-m_y}{\sigma_y}\right) - \rho\left(\frac{x-m_x}{\sigma_x}\right)\right]^2 - \rho^2\left(\frac{x-m_x}{\sigma_x}\right)^2 . \qquad (8)$$

Equation 7 can now be rewritten as

$$I(R) = \int_{x_c-r}^{x_c+r} \exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{x-m_x}{\sigma_x}\right)^2 + \frac{\rho^2}{2(1-\rho^2)}\left(\frac{x-m_x}{\sigma_x}\right)^2\right]$$

$$\left\{\int_{y_c-(r^2-(x-x_c)^2)^{\frac{1}{2}}}^{y_c+(r^2-(x-x_c)^2)^{\frac{1}{2}}} \exp\left[-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{y-m_y}{\sigma_y}\right) - \rho\left(\frac{x-m_x}{\sigma_x}\right)\right\}^2\right] dy\right\} dx . \qquad (9)$$

Define a new integration variable S by:

$$S = \left[\frac{1}{2(1-\rho^2)}\right]^{\frac{1}{2}}\left[\left(\frac{y-m_y}{\sigma_y}\right) - \rho\left(\frac{x-m_x}{\sigma_x}\right)\right] \qquad (10)$$

so that

$$dS = \left[\frac{1}{2(1-\rho^2)}\right]^{\frac{1}{2}} \frac{dy}{\sigma_y} . \qquad (11)$$

From equation 11, one sees that:

$$dy = \sigma_y \left[2(1-\rho^2)\right]^{\frac{1}{2}} dS \ . \tag{12}$$

Now consider what happens to the limits of integration under this change of variable. The upper limit corresponds to $S = S_+$, where

$$S_+(x,r,x_c,y_c) = \left[\frac{1}{2(1-\rho^2)}\right]^{\frac{1}{2}} \left[\left(\frac{y_c+(r^2-(x-x_c)^2)^{\frac{1}{2}}-m_y}{\sigma_y}\right)-\rho\left(\frac{x-m_x}{\sigma_x}\right)\right] \tag{13}$$

The lower limit corresponds to $S = S_-$, where

$$S_-(x,r,x_c,y_c) = \left[\frac{1}{2(1-\rho^2)}\right]^{\frac{1}{2}} \left[\left(\frac{y_c-(r^2-(x-x_c)^2)^{\frac{1}{2}}-m_y}{\sigma_y}\right)-\rho\left(\frac{x-m_x}{\sigma_x}\right)\right] \tag{14}$$

The integral $I(R)$ now becomes:

$$I(R) = \int_{x_c-r}^{x_c+r} \exp\left[-\frac{1}{2}\left(\frac{x-m_x}{\sigma_x}\right)^2\right] \left\{\int_{S_-(x,r,x_c,y_c)}^{S_+(x,r,x_c,y_c)} \exp\left[-S^2\right]\sigma_y\left[2(1-\rho^2)\right]^{\frac{1}{2}} dS\right\} dx \ . \tag{15}$$

The error function, $\mathrm{Erf}(Z)$, is defined by (reference 7, page 297):

$$\mathrm{Erf}(Z) = \frac{2}{\sqrt{\pi}} \int_0^Z e^{-t^2}\, dt \ . \tag{16}$$

In terms of this function, equation 15 is rewritten as:

$$I(R) = \int_{x_c-r}^{x_c+r} \exp\left[-\frac{1}{2}\left(\frac{x-m_x}{\sigma_x}\right)^2\right] \tag{17}$$

$$\left\{\sigma_y\left[2(1-\rho^2)\right]^{\frac{1}{2}} \frac{\sqrt{\pi}}{2}\left[\mathrm{Erf}\left(S_+(x,r,x_c,y_c)\right)-\mathrm{Erf}\left(S_-(x,r,x_c,y_c)\right)\right]\right\}dx \ .$$

Now introduce a new variable, v, so that

$$\left.\begin{aligned} v &= \frac{x-m_x}{\sigma_x} \\ \text{i.e.,}\quad dx &= \sigma_x dv \\ \text{and}\quad x &= v\sigma_x + m_x \end{aligned}\right\} \tag{18}$$

When $x = x_c+r$, $v = (x_c+r-m_x)/\sigma_x$ and when $x = x_c-r$, $v = (x_c-r-m_x)/\sigma_x$. Equation 17 can now be rewritten as:

$$I(R) = \left[2(1-\rho^2)\right]^{\frac{1}{2}}\sigma_x\sigma_y \frac{\sqrt{\pi}}{2}\int_{\frac{x_c-r-m_x}{\sigma_x}}^{\frac{x_c+r-m_x}{\sigma_x}} e^{-\frac{1}{2}v^2} \tag{19}$$

$$\left[\mathrm{Erf}\left(S_+(\sigma_x v+m_x,r,x_c,y_c)\right)-\mathrm{Erf}\left(S_-(\sigma_x v+m_x,r,x_c,y_c)\right)\right]dv.$$

Equation 19 is still exact.

APPROXIMATION OF THE ERROR FUNCTION

In order to simplify equation 19, an approximation for the error function will be used. From reference 7, page 229:

Set $t = 1/(1+px)$ where $p = .3275911$. Then

$$\mathrm{Erf}(x) = 1 - e^{-x^2} \cdot \sum_{i=1}^{5} a_i t^i + \varepsilon(x) \quad . \tag{20}$$

In this equation

$$\left.\begin{array}{ll} a_1 = .254829592 & a_4 = -1.453152027 \\ a_2 = -.284496736 & a_5 = 1.061405429 \\ a_3 = 1.421413741 & \end{array}\right\} \quad . \tag{21}$$

The error term $\varepsilon(x)$ is such that:

$$\varepsilon(x) \le 1.5 \times 10^{-7} \quad . \tag{22}$$

Now set

$$t_{\pm}(v) = \frac{1}{1 + pS_{\pm}(\sigma_x v + m_x, r, x_c, y_c)} \quad . \tag{23}$$

Using equation 23 in equation 19 and then equation 19 in equation 16, leads to the following result.

$$P[R] = \frac{1}{2\sqrt{2\pi}} \int_{\frac{x_c-r-m_x}{\sigma_x}}^{\frac{x_c+r-m_x}{\sigma_x}} e^{-\frac{1}{2}v^2} \left\{ e^{-S_-^2} \cdot \sum_{i=1}^{5} a_i t_-(v)^i \right.$$

$$\left. - e^{-S_+^2} \cdot \sum_{i=1}^{5} a_i t_+(v)^i \right\} dv + e_1 \; . \tag{24}$$

The error term is bounded as follows:

$$|e_1| \le \frac{1}{2\sqrt{2\pi}} 2\varepsilon \int_{\frac{x_c-r-m_x}{\sigma_x}}^{\frac{x_c+r-m_x}{\sigma_x}} e^{-\frac{1}{2}v^2} dv \tag{25}$$

$$\le \frac{1}{\sqrt{2\pi}} (3 \times 10^{-7}) \left(\frac{r}{\sigma_x}\right) . \tag{26}$$

A finer bound on e_1 is:

$$|e_1| \le (3 \times 10^{-7}) \frac{1}{4} \left[\text{Erf}\left(\frac{x_c+r-m_x}{\sqrt{2}\,\sigma_x}\right) - \text{Erf}\left(\frac{x_c-r-m_x}{\sqrt{2}\,\sigma_x}\right) \right] . \tag{27}$$

A SAMPLE CALCULATION

In this section, a sample problem is studied. The following parameters were chosen:

$$\left.\begin{array}{llll} \sigma_x = 200 & \sigma_y = 50 & & \\ m_x = 0 & m_y = 0 & \rho = 0 & . \\ x_c = 0 & y_c = 0 & & \end{array}\right\} \tag{28}$$

These parameters were chosen because the method of reference 6 can be used to obtain $P[R]$ in this case.

According to reference 6, set

$$\sigma_1 = \min(\sigma_x, \sigma_y) \qquad \sigma_2 = \max(\sigma_x, \sigma_y) \quad . \tag{29}$$

Then:

$$P[R] = \frac{\sigma_1}{\sigma_2} \sum_{k=0}^{\infty} \frac{(-1)^k \left(r^2/2\sigma_1^2\right)^{k+1}}{(k+1)!} \left\{ \sum_{i=0}^{k} \left(-\frac{1}{4}\right)^i \binom{k}{i} \binom{2i}{i} \left(1 - \sigma_1^2/\sigma_2^2\right)^i \right\} . \tag{30}$$

Equation 30 is valid only for $m_x = m_x = \rho = 0$.

The calculations were done on a Burroughs 6700 computer. The integral (equation 24) was evaluated by the trapezoid rule. This technique was chosen instead of more advanced schemes, because it is so simple.

In table 1, results obtained using equations 24 and 30 are compared. The integration range in equation 24 was divided into 150 steps. The accuracy of the two calculations is good, even at $r = 275$. However, when the series is used, the amount of computer time used increases rapidly with r. For example, for $r = 250$, it took 4.2 seconds to evaluate the series but only 0.3 seconds to evaluate the integral.

TABLE 1

COMPARISON OF METHODS FOR EVALUATION OF THE COVERAGE INTEGRAL

	P[R]		
r	By Series (30)	By Integral (24)	Percent Difference
25	.030243	.030237	.021
50	.110246	.110222	.022
75	.215952	.215898	.025
100	.325350	.325261	.028
125	.426700	.426570	.031
150	.516830	.516657	.033
175	.596196	.595984	.036
200	.665824	.665578	.037
225	.726484	.726212	.037
250	.778770	.778480	.037
275	.823259	.822962	.036

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APPENDIX

COMPUTER CODE IN THE GENERAL CASE

The following Fortran code can be used to evaluate the coverage integral in the general case.

```
C* THIS SUBROUTINE INTEGRATES THE BIVARIATE NORMAL OVER
C* AN OFFSET CIRCLE.  SX & SY ARE THE STANDARD DEVIATIONS
C* OF THE DISTRIBUTION FUNCTION CENTERED AT UX, UY.  RO
C* IS THE CORRELATION COEFFICIENT. THE INTEGRAL IS EVAL-
C* UATED OVER A CIRCLE OF RADIUS R CENTERED AT X, Y USING
C* THE TRAPEZOIDAL METHOD.  PR CONTAINS THE VALUE OF THE
C* INTEGRAL.
      SUBROUTINE BIVINT(SX,SY,UX,UY,RO,R,X,Y,PR)
      VL=(X-R-UX)/SX
      DL=2.*R/SX/200.
      A1=.254829592
      A2=-.284496736
      A3=1.421413741
      A4=-1.453152027
      A5=1.061405429
      P=.3275911
      SO=SQRT(1./(2.*(1-RO*RO)))
      PR=0.0
      DO 10 J=1,201
      V=(J-1)*DL+VL
      S1=SQRT(R**2-(SX*V+UX-X)**2)
      SU=SO*((Y+S1-UY)/SY-RO*V)
      M1=1
      IF(SU.LT.0) M1=-1
      SU=ABS(SU)
      SL=SO*((Y-S1-UY)/SY-RO*V)
      M2=1
      IF(SL.LT.0) M2=-1
      SL=ABS(SL)
      TU=1./(1.+P*SU)
      TL=1./(1.+P*SL)
      FU=1-(A1*TU+A2*TU**2+A3*TU**3+A4*TU**4+A5*TU**5)*EXP(-SU**2)
      FL=1-(A1*TL+A2*TL**2+A3*TL**3+A4*TL**4+A5*TL**5)*EXP(-SL**2)
      F=EXP(-.5*V**2)*(M1*FU-M2*FL)
      FF=1.0
      IF((J.EQ.1).OR.(J.EQ.201)) FF=0.5
      PR=PR+FF*F*DL
   10 CONTINUE
      PR=SQRT(2.)*PR/(4*SQRT(3.14159))
      RETURN
      END
```

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-8